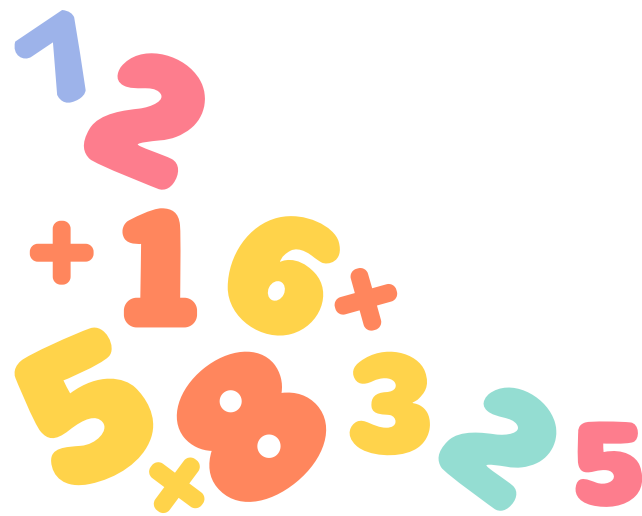


‘Teaching for Mastery’



Maths

Parent Support Booklet



NCETM
NATIONAL CENTRE FOR EXCELLENCE
IN THE TEACHING OF MATHEMATICS

MATHS 
NO PROBLEM!

‘Teaching for Mastery’



Mathematics Mastery Parent Support Booklet

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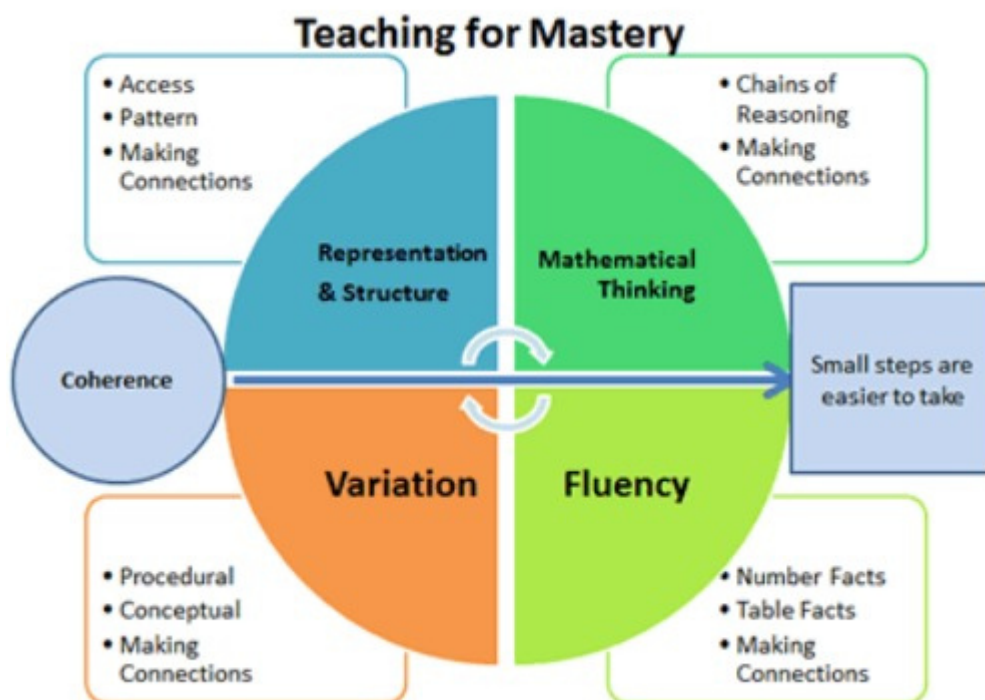
Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject.

The phrase ‘teaching for mastery’ describes the elements of classroom practice and school organisation that combine to give pupils the best chances of mastering maths.

Achieving mastery means acquiring a solid enough understanding of the maths that’s been taught to enable pupils to move on to more advanced material. The [Five Big Ideas](#) underpin teaching for mastery in both primary

National Centre for Excellence in the Teaching of Mathematics
(NCETM)

‘5 Big Ideas’





What do the NCETM '5 Big Ideas' mean?

Coherence

Lessons are broken down into small connected steps that gradually unfold the concept, providing access for all children and leading to a generalisation of the concept and the ability to apply the concept to a range of contexts.

Representation and Structure

Representations used in lessons expose the mathematical structure being taught, the aim being that students can do the maths without recourse to the representation

Mathematical Thinking

If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the student: thought about, reasoned with and discussed with others

Fluency

Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics

Variation

Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the episodes, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to connect the mathematics and draw attention to mathematical relationships and structure.



Concrete, Pictorial and Abstract (CPA Approach)

Concrete, Pictorial, Abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in pupils.

Concrete

Concrete is the “doing” stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects.

Pictorial

Pictorial is the “seeing” stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

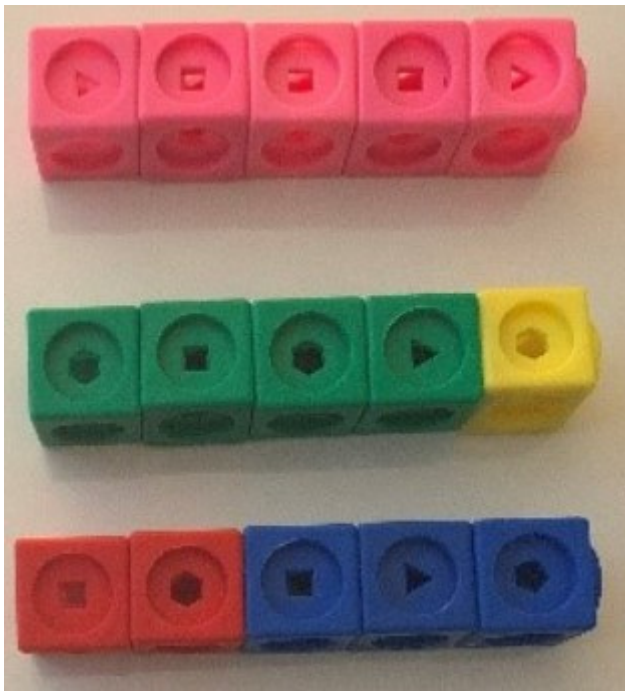
Abstract

Abstract is the “symbolic” stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem.

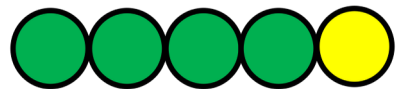
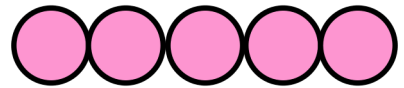


Concrete, Pictorial and Abstract (CPA Approach)

Concrete



Pictorial



Abstract

$$5 + 0 = 5$$

$$4 + 1 = 5$$

$$2 + 3 = 5$$



Examples of Concrete resources

Dienes



Numicon



Dice



Bead String



Cuisenaire Rods



Counters





Representations and Structures

Representations are used in lessons to expose the mathematical structure being taught. They are not new – we can probably all remember using counters when we were children. The history of mathematics uncovers many examples of objects, pictures or symbols used in early maths to represent concepts. Nowadays it's not unusual to find tens frames, Dienes blocks, Cuisenaire® rods, bar models and other representations in frequent use in primary classrooms.

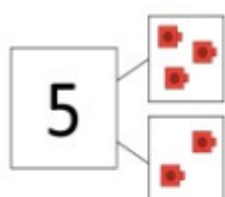
Objects can assist children in performing calculations – for example, a child might use three groups of five counters to then count all the counters to find the product 15. However, using the representations in this way can encourage a child to become dependent on them. Teaching for mastery encourages the use of representations to demonstrate the structure (e.g. three groups of five counters). The child's understanding of the structure is then built on to teach efficient calculation methods.

Representations are useful for all learners, whatever their age. Research mathematicians often use representations to explain their thinking. Teaching for mastery suggests that representations should be used throughout primary to promote a deep understanding of mathematical structure. Once learners have a deep understanding of the maths being represented, the aim is to work with the maths without recourse to the representation, though they will often continue to work with visuals in their mind's eye.

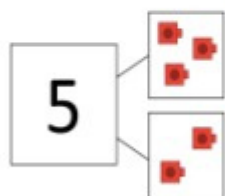


Examples of Representations and Structures

Part Whole Models



The whole is five. I can partition five into one part of three and one part of two.



There are three people in one train carriage and two people in another. One part is three and one part is two. The whole is five.

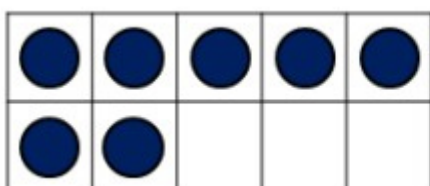
whole = part + part
 $5 = 3 + 2$

Stem Sentences

_____ is the whole; _____ is a part
 and _____ is a part.

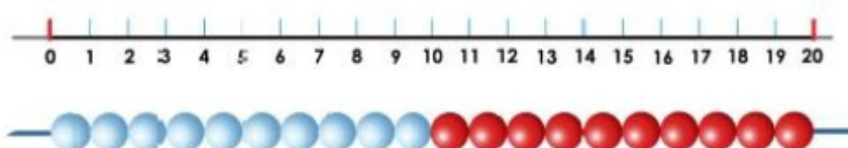
5 is the whole; 3 is a part and 2 is a part.

Ten Frames



*There are seven counters.
 Seven is two more than five.
 Seven is three less than 10.*

Number Lines





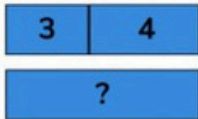
Examples of Representations and Structures


Gattegno Chart

10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

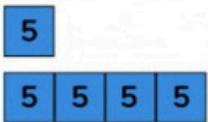
A gattegno chart is a **type of place value chart to help your children with their place value skills**. It's designed to help them appreciate the patterns in the way that we count and our number structure. The way children use a gattegno chart is to count forwards and backwards whilst pointing at the numbers on the chart.

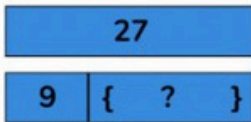
Bar Modelling

ADDITION
 $3 + 4 = ?$

 $3 + 4 = 7$

SUBTRACTION
 $18 - 3 = ?$

 $18 - 3 = 15$

+

MULTIPLICATION
 $4 \times 5 = ?$

 $4 \times 5 = 20$

DIVISION
 $27 \div 9 = ?$

 $27 \div 9 = 3$



SCAN ME

Examples of Vocabulary

What we say	What we mean
Bar Model	This is a way of representing a problem using pictures. It is often a very useful way of making a complex word problem more accessible to pupils. Although it is not in itself a method of solution, by 'seeing' the problem in the visual form, it is then often easier for pupils to see how to approach the problem.
Concrete manipulative	Any physical object that is used to represent a mathematical concept is a concrete manipulative e.g. counters, bead strings, fraction towers, people, straws...The possibilities are endless.
Dienes blocks	Dienes blocks are concrete representations of numbers that are in exact proportion to each other, so they can represent all powers of tens, such as ones, tens, hundreds, thousands; hundredths, tenths, ones and tens; hundreds, thousands, tens of thousands, hundreds of thousands; etc. They help pupils to understand the relationship between place value columns and see why we can exchange e.g. one ten for ten ones.
'Same or different?' tasks	These are useful in developing reasoning: pupils are asked to compare two or more objects, expressions, representations, etc., and asked to identify what they have in common and how they differ.
Skip counting	Selecting a multiple and a starting point and then counting in that multiple, for example, skip counting in fives from one would be 1, 6, 11, 16, 21, 26, 31, etc.
Approximation	The number is not exact but is close, for example, if a journey takes 57 minutes, you might say that it takes approximately one hour.
Commutative	An operation, *, is commutative if for every pair of numbers a and b, a * b = b * a, i.e. the order doesn't matter. Addition and multiplication are commutative, for example, $3 + 4 = 4 + 3$ and $15 \times 65 = 65 \times 15$. Subtraction and division are not commutative.



SCAN ME

Examples of Vocabulary

What we say	What we mean
Factor	A number, that when multiplied with one or more other factors, makes a given number; for example, 2 and 3 are factors of 6 because $2 \times 3 = 6$.
Expression	Numbers, symbols and operators grouped together but without the equal to sign, for example, ' 5×3 or $6 - 1$ '.
Integer	A positive or negative whole number or zero.
Inverse Operation	Two operations are inverses of each other, if when they are combined the number on which they operate, is unchanged. Addition and subtraction are inverse operations, for example, $8 + 9 - 9 = 8$. Multiplication and division are inverse operations, for example, $7 \times 11 \div 11 = 7$.
Number Bond	A way of representing a number using a part-part-whole model; for example, if 3 and 7 are the parts, then the whole is ten.
Partitioning	A way of breaking a number into at least two parts resulting in a number bond for that number, for example, 12 is equal to ten and two.
Prime Number	A whole number that has exactly two factors, itself and one. Examples: 5 (factors 5 and 1), 31 (factors 31 and 1). 57 is not prime (factors 57, 19, 3, 1)".
Subitising	The ability to instantaneously recognise the number of objects in a small group without the need to count them, for example, people generally subitise the number patterns on a die.